subject: Linear Algebra

date: 27 Mar 24

keywords:

topic: 4.1 Systems of Matrices

No class Friday

We get a sub on Monday! Dr. Russell

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 8 & 42 & 7 & -8 \\ 1 & 1 & 1 & \pi \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 2 & 4 & 9 \\ 0 & 0 & 0 & \pi \end{bmatrix} \quad C = \begin{bmatrix} 1 & \pi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Kernel of a matrix: set of vectors that map to $\vec{0}$

A is a "crappy matrix"

B is a row echelon form matrix (REF)

C is a reduced-row echelon form matrix (RREF)

REF – Allows for Back substitution

1) All rows of zeros at bottom
2) First non-zero entry in each row is to the right of first nonzero entry in all rows above

RREF

1) Be REF
2) First non-zero entry in each row is 1
3) All 0's above and below non-zero entries

$$B\vec{x} = \vec{0} \quad \text{where } \vec{x} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$\Updownarrow$$

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 2 & 4 & 9 \\ 0 & 0 & 0 & \pi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Updownarrow$$

$$\begin{aligned} x + y + 2z + 3w &= 0 \\ 2y + 4z + 9w &= 0 \\ \pi w &= 0 \end{aligned}$$

What operations can we do to systems of equ's while preserving sol'ns?

1) Multiply eqn by non-zero scalar
2) Swap position of equations
3) Substitute equation w/ linear combination of other equations

You can do these to a matrix too! "Row Operations"

What about preserving equality?

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 2 & 4 & 9 & 0 \\ 0 & 0 & 0 & \pi & 0 \end{array}\right]$$

Gauss - Jordan Elimination

Pivot!

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & -2 & 4 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Find RREF(A)

$$\xrightarrow{-R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & 1 & -3 \\ 0 & 1 & 1 & 1 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & -4 & 1 & -3 \end{bmatrix} \xrightarrow{4R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 5 & 1 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{5}R_3 \to R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1/5 \end{bmatrix} \xrightarrow{R_2 - R_3 \to R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 4/5 \\ 0 & 0 & 1 & 1/5 \end{bmatrix} \xrightarrow{R_1 - 2R_2 \to R_1} \begin{bmatrix} 1 & 0 & 3 & 12/5 \\ 0 & 1 & 0 & 4/5 \\ 0 & 0 & 1 & 1/5 \end{bmatrix} \xrightarrow{R_1 - 3R_3 \to R_1} \begin{bmatrix} 1 & 0 & 0 & 9/5 \\ 0 & 1 & 0 & 4/5 \\ 0 & 0 & 1 & 1/5 \end{bmatrix}$$

Done! !!