

subject: Linear Algebra

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keywords

topic

A Truly Awful Example

$$T : \mathbb{R}^2 \times \mathbb{P}_1 \to \mathcal{M}_{2\times 2}$$

$$\left(\begin{bmatrix} x \\ y \end{bmatrix}, mx+b\right) \mapsto \begin{bmatrix} x+b & -m \\ y & y-b \end{bmatrix}$$

using $B_{\mathbb{R}^2} = \{e_1, e_2\}$, $B_{\mathbb{P}_1} = \{x+1, x-1\}$

$$\mathcal{B} = B_{Dom} = \{(e_1, x+1), (e_1, x-1), (e_2, x+1), (e_2, x-1)\}$$

$$\mathcal{C} = B_{Codom} = \left\{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right\}$$

Recall: $A = \left[\ \underbrace{[T(e_1, x+1)]_{\mathcal{C}}}_{\vec{b}_1} \ \cdots \ \underbrace{[T(e_2, x-1)]_{\mathcal{C}}}_{\vec{b}_4}\ \right]$

$$T(\vec{b}_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, x+1\right) = \begin{bmatrix} 2 & -1 \\ 0 & -1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 2 \\ 0 \\ -1 \\ -3 \end{bmatrix}$$

$$T(\vec{b}_2) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, x-1\right) = \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

$$T(\vec{b}_3) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}, x+1\right) = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 \\ 1 \\ -1 \\ -2 \end{bmatrix}$$

$$T(\vec{b}_4) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}, x-1\right) = \begin{bmatrix} -1 & -1 \\ 1 & 2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Coord. Vectors

$$A = \begin{bmatrix} 2 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & -2 & 0 \end{bmatrix}$$

For practice $A^T = \begin{bmatrix} 2 & 0 & -1 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & -1 & -2 \\ -1 & 1 & 1 & 0 \end{bmatrix}$

codom $A = \operatorname{col} A \oplus \ker A^T$