subject: Linear Algebra

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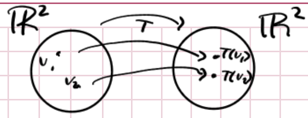
keywords: Matrix Representation, Coordinate Vectors

topic: The Day of Cookies (Also 3.5 More Matrix Stuff)

Example

$T: \mathbb{R}^2 \to \mathbb{R}$ via $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x+2y \\ y-3x \end{bmatrix}$



Note: $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = T\left(x\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = xT\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) + yT\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

$$= \left[T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \;\; T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)\right]\begin{bmatrix} x \\ y \end{bmatrix} = A\vec{x}$$

$$\to A = \left[T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \;\; T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)\right] \in \mathcal{M}_{2\times 2}$$

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix}$$

The Fundamental Theorem of Linear Algebra:
Every linear transformation has an associated matrix representation.

Example Continued

$B = \left\{\underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\vec{b}_1}, \underbrace{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}_{\vec{b}_2}\right\} \subseteq \mathbb{R}^2$

But also $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = T(a\vec{b}_1 + b\vec{b}_2)$

$$= \dots = \left[T(\vec{b}_1) \;\; T(b_2)\right]\begin{bmatrix} a \\ b \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

$$= A\,[x]_B$$

A depends on the basis used

Example 2

$T: \mathbb{P}_2 \to \mathbb{P}_2$ via $ax^2 + bx + c \mapsto (2c-b)x^2 + (a-b)$

$$T(ax^2+bx+c) = aT(x^2) + bT(x) + cT(1)$$

Not column vectors!

Where We're Heading

$$\begin{tikzcd} \mathbb{P}_2 \arrow[r, "T"] \arrow[d, "{[P]_B}"] & \mathbb{P}_2 \arrow[d, "{[P]_B}"] \\ \mathbb{R}^3 \arrow[r, "A"] & \mathbb{R}^3 \end{tikzcd}$$

$B = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$

basis for domain and codomain

$A = \left[\,[T(\vec{b}_1)]_B \cdots [T(\vec{b}_3)]_B\,\right]$

to be continued...