RIGGING NEWTON'S METHOD

(1) Given any function f, we want to find a *root*: some value x such that $f(x) = 0$.

    (a) Write the equation for the line tangent to f at a point $x = x_k$.

    (b) Let x_{k+1} be the x-intercept of the tangent line. Using your answer from part (a), write a formula for x_{k+1} in terms of x_k.

    (c) Newton's Method involves sequentially finding x-intercepts of these tangent lines, using the intercept x_{k+1} as the tangent point to find x_{k+2}, etc. Starting with any guess x_0, will you always find a root? Can you give an example?

(2) Now we *rig* Newton's Method with a function, R.

    (a) Find a function f such that Newton's Method applied to f *always* gives $x_{k+1} = R(x_k)$ (your answer will involve an integral).

    (b) Find f in the case of $R(x) = -x$. What is the behavior of Newton's Method in this case?

(3) The *juggling function* is obtained when $R(x) = 1 - \frac{1}{x}$.

    (a) Prove $R(R(R(x))) = x$ for all x.

    (b) Find f in this case.