CALCULUS II: VOLUME BY SLICING EXAMPLE

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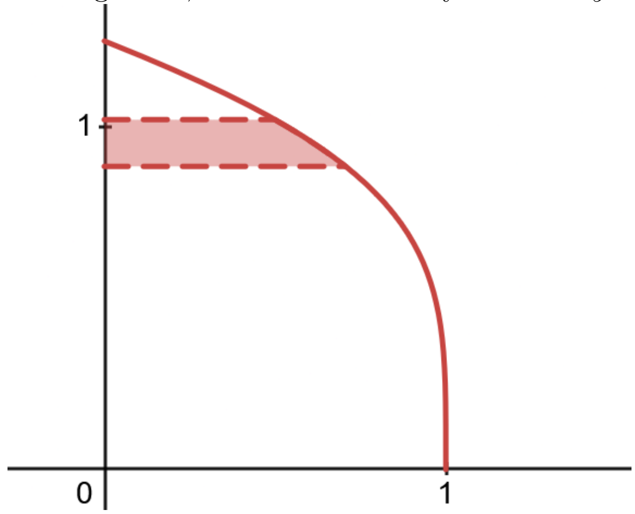
Example 1. Let R be the region bounded by the following curves:

$$y = \sqrt{\cos^{-1} x},\ y = \sqrt{\frac{\pi}{3}},\ y = \sqrt{\frac{\pi}{4}},\ \text{and } x = 0.$$

Use the shell method to find the volume of the solid of revolution when R is revolved around the x-axis.

FIGURE 1. The region R, with the boundary function $y = \arccos^{1/2} x$.



Solution. Solve the above bounds in terms of y:

$$y = \sqrt{\cos^{-1} x} \Rightarrow y^2 = \arccos x \Rightarrow x = \cos y^2$$

Behold! Using the volume by slicing formula for a revolution about the x-axis, we have that

$$V = 2\pi \int_{\sqrt{\pi/4}}^{\sqrt{\pi/3}} y \cos y^2 \mathrm{d}y.$$

Using the u-substitution of $u = y^2$ (so $\mathrm{d}u = 2y\mathrm{d}y$), we have that

$$\begin{aligned} V &= 2\pi \int_{y=\sqrt{\pi/4}}^{\sqrt{\pi/3}} \frac{1}{2} \cos u \mathrm{d}u \\ &= \pi \sin y^2 \Big|_{\sqrt{\pi/4}}^{\sqrt{\pi/3}} \\ &= \pi \left(\sin \frac{\pi}{3} - \sin \frac{\pi}{4} \right) \\ &= \frac{\pi}{2} \left(\sqrt{3} - \sqrt{2} \right). \end{aligned}$$

