I'm seeing a lot of misunderstanding of the Intermediate Value Theorem as I'm grading HW3, so I just want to send this note to explain the theorem and how to apply it.

Theorem 1 (Intermediate Value Theorem)**.** *Suppose $f : [a, b] \to \mathbb{R}$ is a **continuous** function. Then for any N between $f(a)$ and $f(b)$, there exists some number $c \in (a, b)$ with $f(c) = N$.*

To properly use/apply this theorem, you need two facts about your choice of function f. Namely,

1. f must be continuous on your interval $[a, b]$, and
2. Your desired target N must be between $f(a)$ and $f(b)$.

In order to appeal to the IVT, you *must* invoke/state/prove both of these facts. Note: The IVT states that c exists. We do *not* get a formula or recipe for how to find the value of c, or that c is even unique (i.e. there could be several choices of c that have $f(c) = N$).

Example 1. *Show that there is some value for x that achieves*

$$2 + \ln x = \sqrt{x}$$

between 10 and 100.

Proof. Consider the function $f(x) = 2 + \ln x - \sqrt{x}$. We note that f is continuous on the interval $[10, 100]$, since f is the sum of continuous functions 2, $\ln x$, and $\sqrt{x}$. We then compute f on our bounds:

$$f(10) = 2 + \ln 10 - \sqrt{10} \approx 1.14$$

and

$$f(100) = 2 + \ln 100 - \sqrt{100} \approx -3.39.$$

Since 0 lies between $f(10)$ and $f(100)$ (i.e. $-3.39 < 0 < 1.14$), by the Intermediate Value Theorem, there exists some number c such that $f(c) = 0$. Using this fact, we have that

$$\begin{aligned} 0 &= f(c) \\ &= 2 + \ln c - \sqrt{c} \end{aligned}$$

and so

$$\sqrt{c} = 2 + \ln c$$

thus achieving the desired equality in our problem statement. □

Every single sentence of my solution is required for it to be a valid solution. Dropping the statement of continuity means my solution is not correct. Dropping the statement of f evaluated on my bounds, and the 0 is between those bounds, means my solution is not correct. I don't know my value for c analytically, just that some c exists.

I hope this helps!

—Eleanor